**Question 1: (10marks)**

a)

(x + y + z) ‘= x’y’z’

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | y | z | x’ | y’ | z’ | (x+y+z) | **(x+y+z)’** | **x’y’z’** |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | **1** | **1** |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | **0** | **0** |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | **0** | **0** |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | **0** | **0** |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | **0** | **0** |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | **0** | **0** |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | **0** | **0** |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | **0** | **0** |

(xyz)’ = x’ + y’ + z’

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | y | z | x’ | y’ | z’ | xyz | **(xyz)’** | **x’ + y’ + z’** |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | **1** | **1** |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | **1** | **1** |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | **1** | **1** |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | **1** | **1** |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | **1** | **1** |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 | **1** | **1** |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | **1** | **1** |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | **0** | **0** |

b) The distributive law: x + yz = ( x + y )( x + z )

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | y | z | yz | **x + yz** | (x+y) | (x+z) | **( x + y )( x + z )** |
| 0 | 0 | 0 | 0 | **0** | 0 | 0 | **0** |
| 0 | 0 | 1 | 0 | **0** | 0 | 1 | **0** |
| 0 | 1 | 0 | 0 | **0** | 1 | 0 | **0** |
| 0 | 1 | 1 | 1 | **1** | 1 | 1 | **1** |
| 1 | 0 | 0 | 0 | **1** | 1 | 1 | **1** |
| 1 | 0 | 1 | 0 | **1** | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | **1** | 1 | 1 | **1** |
| 1 | 1 | 1 | 1 | **1** | 1 | 1 | **1** |

(c) The distributive law: x(y + z) = xy + xz

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | y | z | (y + z) | **x(y + z)** | xy | xz | **xy + xz** |
| 0 | 0 | 0 | 0 | **0** | 0 | 0 | **0** |
| 0 | 0 | 1 | 1 | **0** | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | **0** | 0 | 0 | **0** |
| 0 | 1 | 1 | 1 | **0** | 0 | 0 | **0** |
| 1 | 0 | 0 | 0 | **0** | 0 | 0 | **0** |
| 1 | 0 | 1 | 1 | **1** | 0 | 1 | **1** |
| 1 | 1 | 0 | 1 | **1** | 1 | 0 | **1** |
| 1 | 1 | 1 | 1 | **1** | 1 | 1 | **1** |

(d) The associative law: x + (y + z) = (x + y) + z

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | y | z | (y + z) | **x + (y + z)** | (x + y) | **(x + y) + z** |
| 0 | 0 | 0 | 0 | **0** | 0 | **0** |
| 0 | 0 | 1 | 1 | **1** | 0 | **1** |
| 0 | 1 | 0 | 1 | **1** | 1 | **1** |
| 0 | 1 | 1 | 1 | **1** | 1 | **1** |
| 1 | 0 | 0 | 0 | **1** | 1 | **1** |
| 1 | 0 | 1 | 1 | **1** | 1 | **1** |
| 1 | 1 | 0 | 1 | **1** | 1 | **1** |
| 1 | 1 | 1 | 1 | **1** | 1 | **1** |

(e) The associative law and x(yz) = (xy)z

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | y | z | yz | **x(yz)** | xy | **(xy)z** |
| 0 | 0 | 0 | 0 | **0** | 0 | **0** |
| 0 | 0 | 1 | 0 | **0** | 0 | **0** |
| 0 | 1 | 0 | 0 | **0** | 0 | **0** |
| 0 | 1 | 1 | 1 | **0** | 0 | **0** |
| 1 | 0 | 0 | 0 | **0** | 0 | **0** |
| 1 | 0 | 1 | 0 | **0** | 0 | **0** |
| 1 | 1 | 0 | 0 | **0** | 1 | **0** |
| 1 | 1 | 1 | 1 | **1** | 1 | **1** |

**Question 2: (10marks)**

**Simplify the following Boolean expressions to a minimum number of literals:**

1. **ABC + A’B + ABC’**
2. **x’yz + xz**
3. **(x + y)’(x’ + y’)**

**(d) xy + x(wz + wz’)**

**(e) (BC’+ A’D) (AB’+ CD’)**

**(f) (a’+ c’) (a + b’+ c’)**

**Question 3: (10 marks) List the truth table of the function: (**

1. **F = xy + xy’ + y’z**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | y | z | x’ | y’ | z’ | xy | xy’ | y’z | **F** |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | **0** |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | **1** |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | **0** |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | **0** |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | **1** |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | **1** |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | **1** |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | **1** |

**(b) F = bc + a’c’**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | c | a’ | b’ | c’ | bc | a’c’ | **F** |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | **1** |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | **0** |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | **1** |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | **1** |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | **0** |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | **0** |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | **0** |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | **1** |

Question 4: (10 marks) Reduce the following Boolean expressions to the indicated number of literals:

**a) A’C’ + ABC + AC’ to three literals**

1. **(x’y’+ z)’+ z + xy + wz to three literals**
2. **A’B(D’ + C’D) + B(A + A’CD) to one literal**
3. **(A’+ C) (A’+ C’) (A + B + C’D) to four literals**
4. **ABC'D + A'BD + ABCD to two literals**

**Logic diagrams are drawn using** [**Visual Paradigm Online**](https://online.visual-paradigm.com/app/diagrams/)

**Image on left is full and on the right simplified version**

**Question 5: (10 marks) Draw logic diagrams of the circuits that implement the original and simplified expressions in Question 2**

1. Full: ABC + A’B + ABC’ Simplified: B

A diagram of a circuit

Description automatically generated A black circle with a blue and black text

Description automatically generated

1. Full: x’yz + xz Simplified: zx +zy

A diagram of a circuit

Description automatically generatedA diagram of a block diagram

Description automatically generated

1. Full: (x + y)’(x’ + y’) Simplified: x’y’

A diagram of a diagram

Description automatically generatedA diagram of a diagram

Description automatically generated

1. Full: xy + x(wz + wz’) Simplified: xy+xw

A diagram of a diagram

Description automatically generatedA black and white diagram

Description automatically generated

1. Full: (BC’+ A’D) (AB’+ CD’) Simplified: 0

A diagram of a circuit

Description automatically generated

1. Full: (a’+ c’) (a + b’+ c’) Simplified: a' b'+c'

A diagram of arrows and arrows

Description automatically generatedA diagram of a circuit

Description automatically generated

Question 6: (10 marks)Draw logic diagrams of the circuits that implement the original and simplified expressions in Question 4

1. Full A’C’ + ABC + AC’ Simplified C’+AB

A diagram of a circuit

Description automatically generatedA diagram of a flowchart

Description automatically generated

1. Full (x’y’+ z)’+ z + xy + wzto Simplified x + y +z

A diagram of a diagram

Description automatically generated with medium confidenceA black line drawing of a flowchart

Description automatically generated

1. Full A’B(D’+ C’D) + B(A + A’CD) Simplified B

A diagram of a diagram

Description automatically generated

1. Full (A’+ C) (A’+ C’) (A + B + C’D) Simplified A' B+A'C'D

A diagram of a person

Description automatically generated A diagram of a circuit

Description automatically generated

1. Full ABC'D + A'BD + ABCD Simplified BD

A diagram of a circuit

Description automatically generated A diagram of a network

Description automatically generated

**Question 7: (10 marks) Simplify the following Boolean functions, using three-variable maps:**

1. F (x, y, z) = Σ (0, 1, 5, 7)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **yz**  **x** | **00** | **01** | **11** | **10** |
| **0** | 1 | 1 | 0 | 0 |
| **1** | 0 | 1 | 1 | 0 |

1. F (x, y, z) = Σ (1, 2, 3, 6, 7)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **yz**  **x** | **00** | **01** | **11** | **10** |
| **0** | 0 | 1 | 1 | 1 |
| **1** | 0 | 0 | 1 | 1 |

1. F(x, y, z)= Σ (2, 3, 4, 5)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **yz**  **x** | **00** | **01** | **11** | **10** |
| **0** | 0 | 0 | 1 | 1 |
| **1** | 1 | 1 | 0 | 0 |

1. F(x, y, z)= Σ (1, 2, 3, 5, 6, 7)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **yz**  **x** | **00** | **01** | **11** | **10** |
| **0** | 0 | 1 | 1 | 1 |
| **1** | 0 | 1 | 1 | 1 |

1. F(x, y, z)= Σ (0, 2, 4, 6)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **yz**  **x** | **00** | **01** | **11** | **10** |
| **0** | 1 | 0 | 0 | 1 |
| **1** | 1 | 0 | 0 | 1 |

1. F(x, y, z)= Σ (3, 4, 5, 6, 7)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **yz**  **x** | **00** | **01** | **11** | **10** |
| **0** | 0 | 0 | 1 | 0 |
| **1** | 1 | 1 | 1 | 1 |

**Question 8: (10 marks) Simplify the following Boolean expressions, using four-variable maps:**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **m** |  | **A** | **B** | **C** | **D** |  | **CD**  **AB** | **00** | **01** | **11** | **10** |
| **0** | **0000** | **A’** | **B’** | **C’** | **D’** | **00** | **M0** | **M1** | **M3** | **M2** |
| **1** | **0001** | **A’** | **B’** | **C’** | **D** | **01** | **M4** | **M5** | **M7** | **M6** |
| **2** | **0010** | **A’** | **B’** | **C** | **D’** | **11** | **M8** | **M9** | **M11** | **M10** |
| **3** | **0011** | **A’** | **B’** | **C** | **D** | **10** | **M12** | **M13** | **M15** | **M14** |
| **4** | **0100** | **A’** | **B** | **C’** | **D’** |
| **5** | **0101** | **A’** | **B** | **C’** | **D** |
| **6** | **0110** | **A’** | **B** | **C** | **D’** |
| **7** | **0111** | **A’** | **B** | **C** | **D** |
| **8** | **1000** | **A** | **B’** | **C’** | **D’** |
| **9** | **1001** | **A** | **B’** | **C’** | **D** |
| **10** | **1010** | **A** | **B’** | **C** | **D’** |
| **11** | **1011** | **A** | **B’** | **C** | **D** |
| **12** | **1100** | **A** | **B** | **C’** | **D’** |
| **13** | **1101** | **A** | **B** | **C’** | **D** |
| **14** | **1110** | **A** | **B** | **C** | **D’** |
| **15** | **1111** | **A** | **B** | **C** | **D** |

1. A’B’C’D’+ AC’D’+ B’CD’+ A’BCD + BC’D

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **CD**  **AB** | **00** | **01** | **11** | **10** |
| **00** | 1 |  |  | 1 |
| **01** |  | 1 | 1 |  |
| **11** | 1 | 1 |  |  |
| **10** | 1 |  |  | 1 |

(b) xz + w xy + w(xy + xy) = xz +wxy

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **wz**  **xy** | **00** | **01** | **11** | **10** |
| **00** |  |  |  |  |
| **01** |  | 1 | 1 |  |
| **11** |  | 1 | 1 | 1 |
| **10** |  |  |  |  |

(c) ABCD + ABD + ABC + ABCD + ABC

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **CD**  **AB** | **00** | **01** | **11** | **10** |
| **00** |  |  |  |  |
| **01** |  |  |  |  |
| **11** |  | 1 | 1 | 1 |
| **10** |  |  |  |  |

1. ABCD + BCD + ACD + ABCD + ACD=BCD+ACD



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **CD**  **AB** | **00** | **01** | **11** | **10** |
| **00** |  |  |  |  |
| **01** |  |  | 1 |  |
| **11** |  |  | 1 |  |
| **10** |  |  | 1 |  |

**Question 9: (10 marks) Find the minterms of the following Boolean expressions by first plotting each function in a map:**

(a) xy + yz + xy’z

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **yz**  **x** | **00** | **01** | **11** | **10** |
| **0** |  |  | 1 |  |
| **1** |  | 1 | 1 | 1 |

1. C’D + ABC’+ ABD’+ A’B’D

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **CD**  **AB** | **00** | **01** | **11** | **10** |
| **00** |  | 1 | 1 |  |
| **01** |  | 1 |  |  |
| **11** | 1 | 1 |  | 1 |
| **10** |  | 1 |  |  |

(c) wyz+ w’x’+ wxz’

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **yz**  **wx** | **00** | **01** | **11** | **10** |
| **00** | 1 | 1 | 1 | 1 |
| **01** |  |  |  |  |
| **11** | 1 |  | 1 | 1 |
| **10** |  |  | 1 |  |

1. A’B + A’CD + B’CD + BC’D’

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **CD**  **AB** | **00** | **01** | **11** | **10** |
| **00** |  |  | 1 |  |
| **01** | 1 | 1 | 1 | 1 |
| **11** | 1 |  |  |  |
| **10** |  |  | 1 |  |

**Question 10**

**Convert the following Boolean function from a sum-of-products form to a simplified product-of-sums form. F(w, x, y, z) = Σ (0, 1, 2, 5, 8, 10, 13)**



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **yz**  **wx** | **00** | **01** | **11** | **10** |
| **00** |  |  | 0 |  |
| **01** | 0 |  | 0 | 0 |
| **11** | 0 |  | 0 | 0 |
| **10** |  | 0 | 0 |  |